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Realization and dissemination of units in a revised SI

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ABSTRACT

The General Conference of Weight and Measures (CGPM) with its Resolution 12 has officially prospected the adoption, possibly at the next General Conference of 2011, of new definitions for kilogram, ampere, kelvin and mole using fundamental constants as reference quantities. Thus, the outlines and features of the SI after that date are worthy to be considered and a general afterthought of the metrological activity in the new situation to be attempted.

The future realisations of units will greatly benefit from the accelerated endeavour to determine the relevant fundamental constants with the minimum possible uncertainty before their values are fixed by definition. In fact, many of those experiments will become realisations of units when the new definitions are adopted.

To define the units by reference to fundamental constants implies to abandon the identification of the unit with its primary standard, as in the old metrological tradition. To realise a unit will definitely consist in assigning a value to a primary standard, consistent with the fixed values of the reference constants, by means of an experimental procedure, independent of a specific definition, which could even not exist. The primary standard should be suitable to dissemination by direct comparison, thus essentially stable and accessible with the highest precision, while the role of the realisation experiment would be mainly related to indirect measurements, typical of scientific activity, which involves the coherence of the unit system. The two distinct roles, of unit realisation and primary standard, correspond to different uncertainty components, of which only one is implied in dissemination activity, just aiming at compatibility of measurements of a specific quantity. Each of the two uncertainty components has a different evolution from the time of the unit redefinition.

These last considerations could validly contribute in critical issues, such as deciding whether the time for a unit redefinition has come or it should be preferable to wait for new and better experiments before fixing the value of a constant. This could be the case for the kilogram redefinition.

1. Introduction

In the last years, the proposals and actions [1–3] intended to redefine some units of the International System (SI) have produced a remarkable acceleration toward a system based on fundamental constants. This trend started with the ampere definition by reference to the magnetic constant μ_0 and continued with the redefinition of the metre based on the velocity of light in vacuum c.

Fundamental constants, being universal and invariant quantities, are appropriate references for measurement units. Among other physical invariants with no general role in scientific theories, such as a parameter of a specific atom, they give the advantage that fixing exactly the values of a suitable subset of them significantly reduces the uncertainties of many other related constants, thus resulting in a great impact on the scientific use of the SI [1].

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The initial proposal [1] was supported by the International Committee for Weights and Measures (CIPM) [2], which recommended the issue for consideration to all the relevant Consultative Committees and institutional bodies. At last, the most official acknowledgment of the proposed redefinitions was the Resolution 12 of the 23rd General Conference on Weights and Measures (CGPM), which "recommends that National Metrological Institutes and the BIPM pursue the relevant experiments so that the International Committee can come to a view on whether it may be possible to redefine the kilogram, the ampere, the kelvin and the mole using fixed values of the fundamental constants at the time of the 24th General Conference (2011)" [4].

2. The revised SI in the leading proposals

Following the proposal in [1] and taking into account also other significant proposals, the possible substitutions for the present reference quantities are shown in Fig. 1, where the links among the basic units introduced by the definitions are omitted.

Together with the first proposal, detailed and thoroughly discussed in [3], a more radically renewed unit system has been outlined in the same paper, which would overcome all problems concerning the more or less involved wording of definitions, mainly caused by the noncoincidence between the base units and the units in which the fixed constants are expressed.

In the proposed system, the distinction between basic and derived units is abolished and a unique synthetic definition is adopted for any SI unit, without associating a particular unit with a particular constant: the SI units are such that the fixed constants have the specified values when expressed in those units.

If the proposed redefinitions are adopted while maintaining the present hierarchical structure of the system with basic and derived units, the unit $[Q_i]$ of any quantity Q_i will be expressed as a function of the seven base units $[Q_B]$ as

$$[Q_i] = \prod_{h=1}^{7} [Q_{Bh}]^{p_{hi}} \tag{1}$$

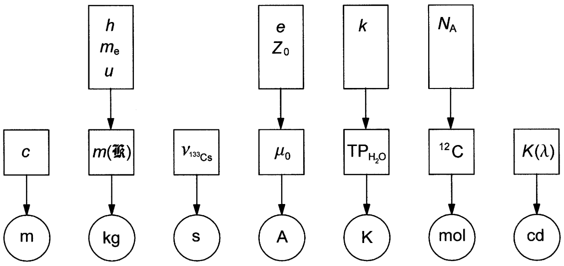

Fig. 1. The present reference quantities for the basic SI units and their substitutions with fundamental constants following the most remarkable proposals. For the kilogram: beside h, also the electron mass m_e or the atomic mass unit u could replace the mass of the International Prototype; for the ampere: beside e, also the impedance of free space Z_0 could substitute for the magnetic constant μ_0; for the kelvin, the Boltzmann constant k could be the reference instead of the triple point of water PT_{H_2O}; for the mole, the Avogadro constant N_A is proposed instead of ^{12}C.

Eq. (1) can be also seen as the well known dimensional analysis of the unit $[Q_i]$.

On the other side, any base unit $[Q_B]$ can be expressed as a function of one or some of those constants which are adopted as the reference of the system. Namely, for $h = 1, 2, \ldots, 7$, it is

$$[Q_{Bh}] = \prod_{k=1}^{7} \left(\frac{C_k}{\{C_k\}} \right)^{p_{kh}} \tag{2}$$

In general, the base units do not coincide with the units $C_k/\{C_k\}$ of the relative reference constants, so the definitions may involve also other units. However, expressing them with basic units as in Eq. (1) yields a set of equations of which (2) are the solutions. Thus, keeping into account Eq. (2), Eq. (1) becomes

$$[Q_i] = \prod_{k=1}^{7} \left(\frac{C_k}{\{C_k\}} \right)^{p_{ki}} \tag{3}$$

In case the more advanced approach outlined above would be adopted, with the abolition of the basic units, Eq. (3) could be written directly, without the intermediation of Eqs. (1) and (2).

Thus, the two approaches are coincident in the substance and the units $C_k/\{C_k\}$ could be seen as the actual dimensions in a system entirely referred to constants.

3. Realization of units in a system based on fundamental constants

Whichever of the above proposals will be adopted, the present experiments for the determination of the reference constants could become realizations of units in the new situation. As an example, the system for the determination of the Planck constant by reference to mass and kinematical quantities, namely the watt balance, once the kilogram would be redefined on the basis of a fixed value of h could be used for the realization of that unit.

In general, to realize a unit means to assign a value to a measurement standard by means of an experiment that establish a relation with one or more of the fixed constants, involving also other quantities if necessary.

Let

$$S = \{S\}[Q_S] \tag{4}$$

be the standard to which the numerical value $\{S\}$ is to be assigned in order to realize the unit $[Q_S]$ complying with condition (3), and let

$$S = f(C_k, Q_i), \tag{5}$$

with k any value(s) from 1 to 7 and i extended to all SI quantities, be the mathematical model of the experiment as a function of one or more of the seven fixed constants C_k and of possible other SI quantities Q_i . Then the very objective of the experiment is the determination of $\{S\}$ in expression (4) as a function of $\{C_k\}$ and $\{Q_i\}$, that is

$$\{S\} = f(\{C_k\}, \{Q_i\}). \tag{6}$$

If the function implied in the model of an experiment, like (5), is such that the standard S can be related to the cur-

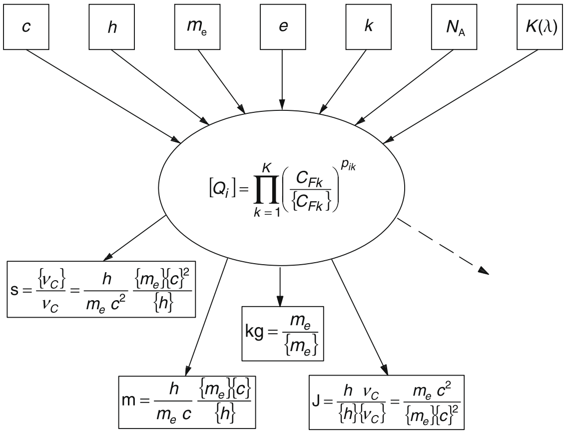

Fig. 2. Diagram showing with some example how starting from a given set of fundamental constants it is possible to realize any SI unit, base or derived, through Eq. (3), which underlies all types of definition. The set of fixed constants is one of those included in Fig. 1. As a possible reference quantity for the time unit, the period corresponding to the Compton frequency of electron ν_C is assumed.

rently fixed constants, then the experiment is commonly accepted as a realization of the unit $[Q_S]$ even in the present situation, independently of the specific definition now in force. In fact, a measurement unit is usually realized through the most convenient experimental means, provided its equivalence to the SI definition can be demonstrated.

The diagram of Fig. 2 shows as, in substantial terms, the units can be realized in an SI based on fundamental constants. Once again, one can see how much the more advanced hypothesis of a unique synthetic definition for all units, fixing the set of reference constants with exact values, would be coherent with the most essential aspect of unit realization.

As for the uncertainty of realization, its components results from the uncertainty with which the model represents the real measuring system, in particular the uncertainty of definition of the measurand and the measurement uncertainties of quantities Q_i, if any, propagated through the function. As for the numerical values $\{C_k\}$, they are to be considered exact by definition.

4. System coherence and measurement compatibility

The dissemination process develops through calibration of standards at different hierarchical levels, which essentially consists in the determination of the value of a standard from the value of the upper level standard and from the measurement of the small difference between the two standards.

For the calibration of a generic standard, following backward its traceability line up to the primary standard S, one can write

$$\{S'_n\} = \{S\} + \{S_1 - S\} + \sum_{i=2}^{n}\{S'_i - S'_{i-1}\} = \{S\} + \{\Delta S'\} \qquad (7)$$

and for the measurement value of a given measurand Q, using S'_n as reference quantity,

$$M'(Q)/[Q] = \{S'_n\} + \{Q - S'_n\} = \{S\} + \{\Delta S'\} + \{\Delta Q'\}. \qquad (8)$$

In order to analyse the compatibility of measurements performed within the metrological structure having S as its vertex, consider measuring the same measurand Q at a different point of the traceability tree, using S''_m as reference quantity. For such a measurement it is

$$\begin{aligned} M''(Q)/[Q] &= \{S''_m\} + \{Q - S''_m\} \\ &= \{S\} + \{\Delta S''\} + \{\Delta Q''\}, \end{aligned} \qquad (9)$$

$\{\Delta S''\}$ being the sum of m steps along the different traceability line. By inspection, the difference of the two measurements reduces to a sum of measured differences along a closed loop including S and Q. This sum is ideally zero, while in reality it depends on the uncertainties of the difference measurements and on the stabilities of S, Q and the intermediate involved standards.

However, it is important to conclude that the compatibility of the two measurements is independent of the value $\{S\}$ and of its realization uncertainty, but only depends on its variations during the dissemination process. In other words, the realization uncertainty of S does not affect the difference of the two measurements, being a highly correlated uncertainty component.

Within a given quantity, the traceability to a standard at whatever hierarchical level warrants the compatibility of all measurements traceable to that standard. Traditionally, the traceability to a national standard has been the basis for the measurement compatibility within a country. In some cases, mutual recognition agreements extended the compatibility to groups of two or more countries. Today, the Mutual Recognition Arrangement (RMA) makes measurement compatibility hold on a regional or a potentially global basis.

In an ideal implementation of MRA project, several national metrological laboratories realize the units with independent experiments, the primary standards are intercompared and a single reference value is evaluated. Therefore, the reference value can be seen as the real vertex of the hierarchy of the standards and, when derived from independent experiments, also the best realization of the unit, although a virtual standard.

In principle, as the ultimate exit of an international comparison, the outstanding features of the reference value could be extended back to the national standards reassigning their values with a reduced uncertainty (provided the quality of the standards is sufficient). Such a procedure would result in a progressive alignment of the local references for the unit dissemination and in a better compatibility of all measurements.

Indirect measurements are essential for science, whose principal aim is to establish relations among quantities. Also in metrology, they are widely used to measure a quantity for which a specific standard is not available.

The compatibility of an indirect measurement with a direct measurement of the same measurand can be analysed with a procedure similar to the one followed above.

Let be

$$Q = Q_1^{p_1} Q_2^{p_2} \qquad (10)$$

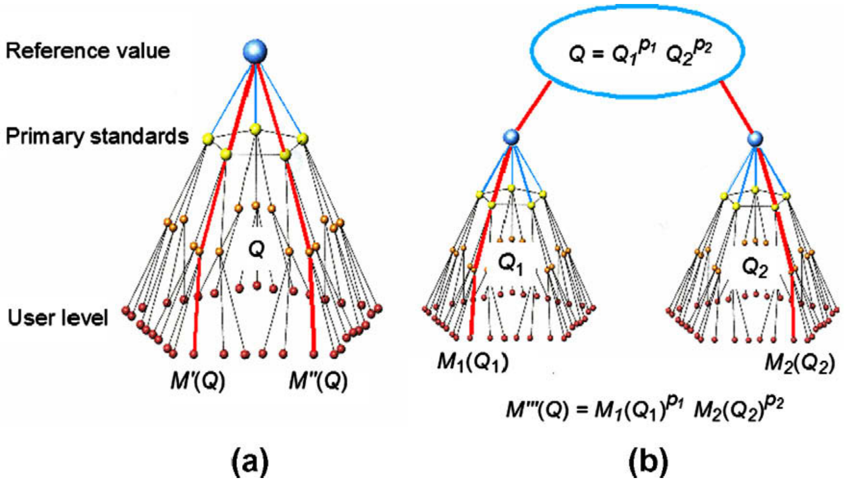

Fig. 3. Direct and indirect measurement in a metrological structure organized on a global basis. Reference quantities at different hierarchical levels are connected by dissemination lines which become traceability lines, convergent to a virtual reference quantity whose value is the reference value of an MRA exercise: (a) within a single quantity, the compatibility of direct measurements $M'(Q)$ and $M''(Q)$ traceable to a common standard is independent of the unit realization uncertainty and (b) for compatibility with an indirect measurement $M'''(Q)$, also coherence of units and realization uncertainty is implied.

the relation on which the indirect measurement is based. Let also be

$$\begin{aligned} &M'(Q)/[Q] = \{S\}(1+\delta'),\\ &M(Q_1)/[Q_1] = \{S_1\}(1+\delta_1),\\ &M(Q_2)/[Q_2] = \{S_2\}(1+\delta_2) \end{aligned} \tag{11}$$

the direct measurements of quantities $Q,\ Q_1$ and Q_2.

From Eq. (11) the indirect measurement of the same measurand can be calculated as

$$M'''(Q) = \{S_1\}^{p_1}\{S_2\}^{p_2}(1+\delta''') \tag{12}$$

where δ''' is a function of δ_1 and δ_2.

The sufficient conditions for being $M'''(Q) = M'(Q)$, as ideally requested by compatibility, are

$$\delta' = \delta''' \quad \text{and} \quad \{S\} = \{S_1\}^{p_1}\{S_2\}^{p_2}. \tag{13}$$

The further condition on units $[Q] = [Q_1]^{p_1}[Q_2]^{p_2}$ is exactly met for the coherence of the system of units.

The first of conditions (13) regards the dissemination process and the direct measurements of the involved quantities and is met within the relevant uncertainties. The second condition does not depend on dissemination, but is met within an uncertainty which propagates from the realization uncertainties of the three involved units.

Thus, different from the direct measurements, indirect measurements are affected by the realization uncertainties of the relevant units, which in general are to be considered as uncorrelated.

The difference between direct and indirect measurements is put in evidence in Fig. 3 in its aspects concerning measurement compatibility within metrological structures.

5. Uncertainty statement in the new situation

The main objective of a unit realization, calibration or measurement is to find the value to be assigned to a standard or a measurand. Nevertheless, it is well known that the statement of any metrological result is incomplete until the unit and the uncertainty are specified. As a matter of clarity, the unit should be thought as an exact entity, defined within a specified system, while the uncertainty affects the value, which could be thought rather as a distribution of value, than a single number.

In a unit system where the units are defined by reference to practical standards, as it typically was in the past, the realization uncertainty essentially coincided with the stability and reproducibility features of the primary standard. At the limit, that uncertainty is null when an individual standard is identified by definition as the unit, as still it is for the present SI unit of mass. Such a unit system is rather suitable to trade uses or even to some technical purposes, than to a scientific use, where the reference to physical invariants is more useful.

In the revised SI, once the new definitions are adopted, no unit will be defined by reference to an individual standard, so that any primary standard will be affected by a realization uncertainty and their value subject to periodical adjustment.

It may happen that the performance of some primary standard exceeds the accuracy of the value assigned realizing the unit in its new definition, so that one could put the question whether the time for redefinition has come or it should be convenient to wait for new and better experiments before fixing the value of a constant. This is the case for the unit of mass, where the accuracy of a realization referred to the Planck constant would suffer by a discrepancy between two different methods of determinations of that constant.

Immediately after redefinition, the value of the standard, namely 1 kg, assigned as a realization of the unit will be the same as with the present definition, for the criterion of continuity, but it would take the relative uncertainty which was that of the reference constant. When a new realization of the unit is performed, presumably with a re-

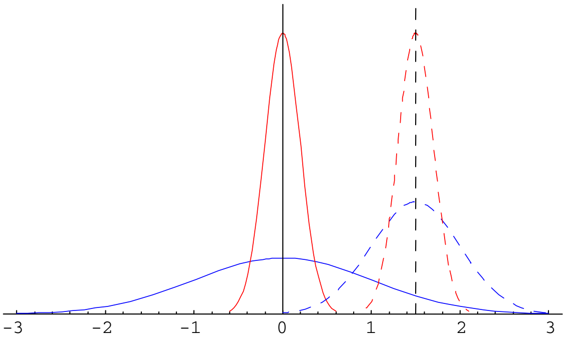

Fig. 4. Possible evolution of the two main uncertainty components of a primary standard after redefinition. The wider (blue) curves represent the component u_{SI} related to the SI unit realization, the narrower (red) diagrams refer to the component u_S depending on the performance of the standard. The dashed diagrams show the possible situation when a new realization of the unit is performed. In abscissa is the deviation, in arbitrary scale, from the centre value after redefinition. (For interpretation of references in color in Fig (4), the reader is referred to the web version of this article.)

duced uncertainty, the value of the standard could change. In this case, it should be accepted that also the value once invariable by definition be subject to adjustment, as any other standard.

Fig. 4 shows the possible situation with reference to the two main components of uncertainty: the one concerning the SI unit realization (u_{SI}) and the other related to the performance of the primary standard (u_S).

As considered above, u_S is the only component involved in dissemination activity, which could be prosecuted, in a first period after the unit redefinition, with the same standards and same values as before. As for the uncertainty statement in a calibration or measurement result, it seems convenient that both u_S and u_{SI} components be specified, so that they can be properly kept into account by the user in any uncertainty budget. This will also prevent a non-SI unit based on the standard from replacing the SI unit referred to the fixed constants.

Thus, as far as direct measurements and compatibility within a given quantity are regarded, only u_S component is concerned. Where indirect measurements involve independent quantities, also u_{SI} component is relevant.

A similar situation has been experienced after 1990 for electrical measurements, even if the emphasis then given to exactness of the constants K_{J-90} and R_{K-90} was equivalent to the institution of an independent system for all electrical units.

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